Hale School
2010

Question/Answer Booklet

MATHEMATICS 3CD
Section One
(Calculator Free)

Circle your teacher's initials

GJ JIB BAH

Your name: SOLUTIONS

Time allowed for this section

Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this examination

	Number of questions	Working time (minutes)	Marks available
This Section (Section 1) Calculator Free	***6***	***50***	***40***
Section Two Calculator Assumed	11	100	80
		Total marks	120

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

4. Show all working clearly. Any question, or part question, worth more than 2 marks requires valid working or justification to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

Question 1 (6 marks)

a) Solve for x: $\dfrac{2x^2+1}{x-1}+3=0$

Critical values: $x \neq 1$ and

$2x^2+1=-3(x-1)$

$2x^2+3x-2=0$

$(2x-1)(x+2)=0$

$x=\frac{1}{2}$ or $x=-2$

[3]

b) Simplify: $\dfrac{x}{x-3}+\dfrac{2x+1}{x+2}$

[3]

$=\dfrac{x(x+2)+(2x+1)(x-3)}{(x-3)(x+2)}$

$=\dfrac{x^2+2x+2x^2-5x-3}{(x-3)(x+2)}$

$=\dfrac{3x^2-3x-3}{(x-3)(x+2)}$

Question 2 (8 marks)

The following system of equations does not have a unique solution

$x + y + pz = 3$ ① $\quad 3x - y - z = p$ ② $\quad x + 5y + 9z = 11$ ③

a) Show that there cannot be an infinite number of solutions

[5]

③ − ① : $4y + (9-p)z = 8$ —— ④

② − 3③ : $-16y - 28z = p - 33$ —— ⑤

4④ + ⑤ : $(36 - 4p - 28)z = 32 + p - 33$

$(8 - 4p)z = p - 1$

For infinite solutions, $0z = 0$

$8 - 4p = 0$ and $p - 1 = 0$

$p = 2$ $\quad p = 1$

As it is not possible for p to be 1 and 2 at the same time, this system cannot have infinite solutions.

b) Hence, determine the value of p so that the system has no solutions.

[3]

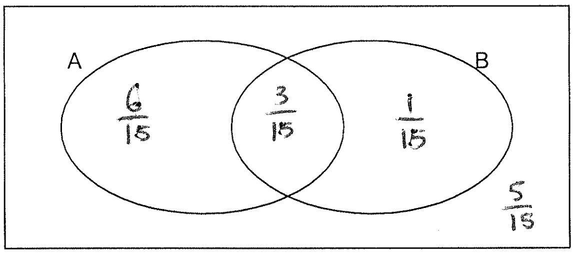
For no solutions, $0z = k$, where $k \neq 0$

∴ for no solutions, $p = 2$ (and $p \neq 1$).

Question 3 **(8 marks)**

For events A and B represented in the Venn Diagram below:

$P(A \cap B) = 0.2$ $\quad P(A) = 0.6$ $\quad P(A | B) = 0.75$



a) Find: [4]

(i) P(B) $= \dfrac{P(A \cap B)}{P(A|B)} = \dfrac{0.2}{0.75} = \dfrac{4}{15}$

(ii) $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$

b) Are the events A and B independent? Justify your answer. [2]

If independent $P(A) \times P(B) = P(A \cap B)$

$\frac{9}{15} \times \frac{4}{15} \neq \frac{3}{15}$

∴ Events A & B are <u>not</u> independent.

c) Are the events A and $\overline{B}$ independent? Justify the answer. [2]

$P(A) \times P(\bar{B})$

$= \frac{9}{15} \times \frac{11}{15}$

$= \frac{11}{25}$

$\neq P(A \cap \bar{B})$

$P(A) = \frac{9}{15}$

$P(B) = \frac{4}{15}$

$P(\bar{B}) = \frac{11}{15}$

$P(A \cap \bar{B}) = \frac{6}{15}$

A and $\bar{B}$ <u>not</u> independent

Question 4 (4 marks)

Explain clearly why the following set of equations has ~~no~~ infinite solutions:

$$\begin{aligned} -x - 3y - 3z &= -7 \quad \text{—— ①} \\ 3x + 9y + 9z &= 21 \quad \text{—— ②} \\ -4x - 2y + 2z &= 24 \quad \text{—— ③} \end{aligned}$$

Note that rows 1 and 2 are the same planes.

This does not give a unique solution

Question 5 **(10 marks)**

a) Find $f'(x)$ giving each answer in simplest form using positive indices.

(i) $f(x)=\sqrt{5-x^4}$ [2]

$$f'(x) = \frac{1}{2}(5-x^4)^{-\frac{1}{2}} \cdot 4x^3$$

$$= \frac{2x^3}{\sqrt{5-x^4}}$$

(ii) $f(x)=(x-2)^4x^2$ [3]

$$f'(x) = 4(x-2)^3(x^2) + (x-2)^4 \cdot 2x$$

$$= 2x(x-2)^3\left[2x + (x-2)\right]$$

$$= 2x(x-2)^3(3x-2)$$

(iii) $f(x)=\dfrac{x}{(x+1)}$ [2]

$$f'(x) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

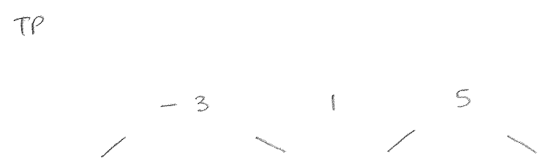
b) Evaluate: $\dfrac{d}{dx}\left(\dfrac{tx^2}{x^2+2x-1}\right)$ [3]

$$= \frac{2tx(x^2+2x-1) - (tx^2)(2x+2)}{(x^2+2x-1)^2}$$

$$= \frac{-2tx}{(x^2+2x-1)^2}$$

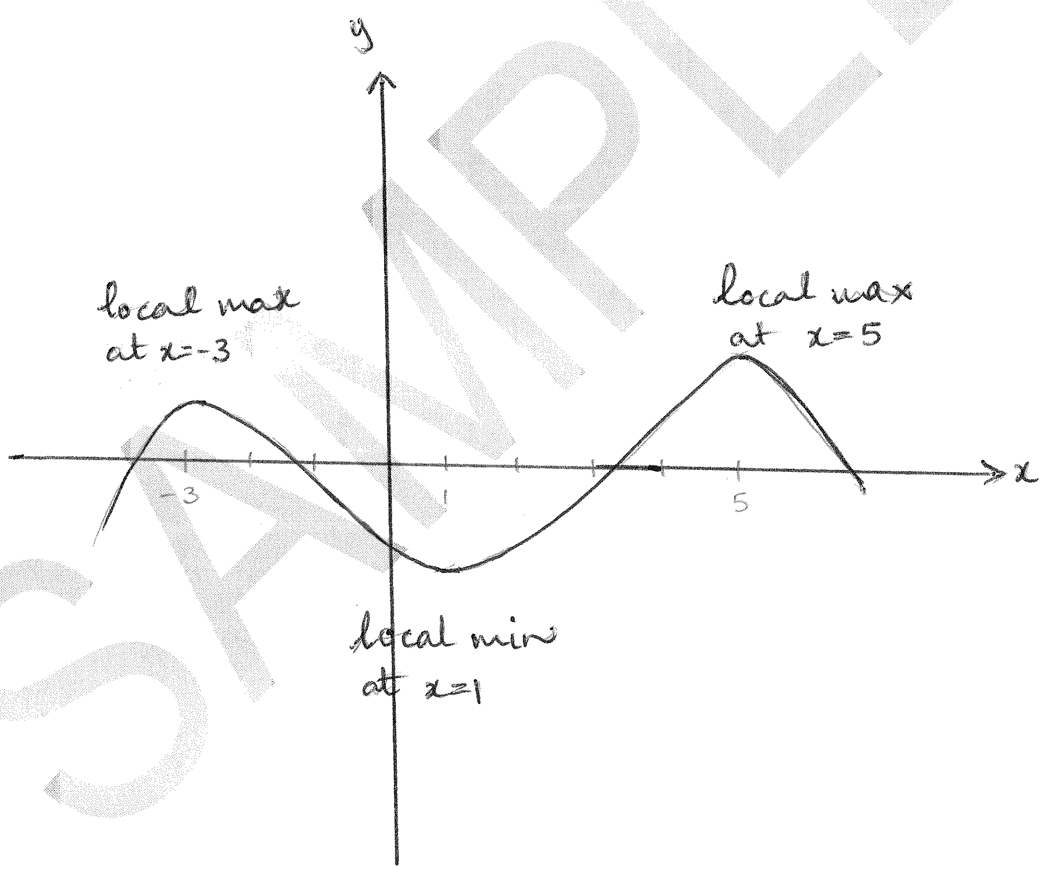
Question 6 (4 marks)

For a particular function $y = f(x)$



- $\frac{dy}{dx} = 0$ at $x = -3$, 1 *and* 5
- $\frac{dy}{dx} > 0$ *when* $x < -3$ *and* $1 < x < 5$
- $\frac{dy}{dx} < 0$ *when* $-3 < x < 1$ *and* $x > 5$

Sketch a possible graph to incorporate all of these features.



Question 7 **(8 marks)**

a) A discrete random variable $\mathbf{X}$ can take on values 0 , 1 or 2:

$$P(X = x) = \frac{\binom{5}{x}\binom{2}{2-x}}{\binom{7}{2}} \quad \text{for } x = 0, 1 \text{ or } 2$$

i) Show, in the form of a table, the probability distribution for $\mathbf{X}$ [3]

x	0	1	2		
$P(X=x)$	$\frac{1}{21}$	$\frac{10}{21}$	$\frac{10}{21}$		

ii) Find $P(X < 1 \mid X \leq 2)$ $= \frac{1}{21}$ [1]

b) A continuous random variable $\mathbf{Y}$ has the triangular probability density function *f:*

$$f(y) = \begin{cases} 0 & \text{for } y < 0 \\ \mathbf{b}(10 - y) & \text{for } 0 \leq y \leq 10 \\ 0 & \text{for } y > 10 \end{cases}$$

i) Find the constant ***b.*** [2]

$\int_0^{10} b(10-y)\, dy = 1$

$b = \frac{1}{50}$

ii) Find $P(\mathbf{Y} \geq 1)$) [2]

$1 - \int_0^1 0.02\,(10-y)\, dy = 0.81$ (2 dp)

Question 8 ***(8 marks)***

3 boarding housemasters and 8 day house masters are to sit in the front row at assembly.

a) In how many ways can they be arranged in a row

(i) without restriction?

[1]

$11! = 39916800$

(ii) if a boarding housemaster must be on each end of the row?

[2]

$3 \times 9! \times 2$

$= 2177280$

(iii) if the boarding housemasters must sit together?

[2]

$3! \times 9!$

$= 2177280$

b) Find the probability that the Buntine housemaster must ***not*** sit next to the Parry housemaster.

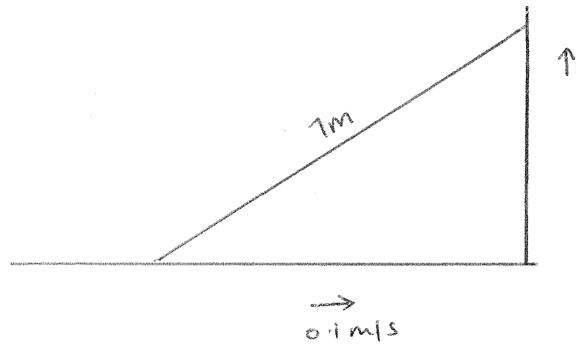
[3]

$1 - \frac{10! \times 2}{11!}$

$= 0.\overline{81}$

Question 9 **(5 marks)**

A ladder 7m long rests against a vertical wall and is standing on flat ground. The bottom of the ladder is being pulled along the ground and towards the wall at a steady rate of 0.1 m/s. How fast is the top sliding up the wall when the bottom is 2m out of the wall?



$$\frac{dx}{dt} = -0.1 \text{ m/s}$$

$$y = (49 - x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(49 - x^2)^{-\frac{1}{2}} \cdot -2x$$

$$= \frac{-x}{\sqrt{49 - x^2}}$$

$$\frac{dy}{dt} = \frac{-x}{\sqrt{49 - x^2}} \times -0.1$$

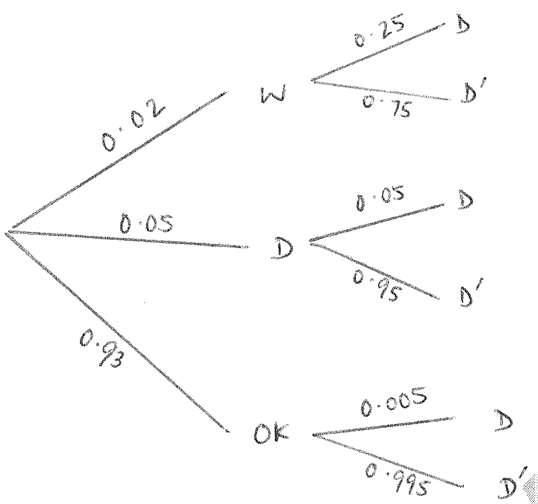
$$\left.\frac{dy}{dt}\right|_{x=2} = \frac{-2}{\sqrt{49 - 4}} \times -0.1$$

$$= 0.0298 \text{ m/s} \quad (4 \text{ dp})$$

Question 10 *(11 marks)*

The foreman in a brick factory knows that the sand used for casting is too dry 5% of the time, and too wet 2% of the time.
He also knows that defects occur 0.5% of the time when the sand is perfect, 5% when sand is too dry, and 25% of the time when the sand is too wet.

a) Represent the above information using a probability tree and showing all relevant probabilities. [3]



P(WD) = 0·005

P(WD') = 0·015

P(DD) = 0·0025

P(DD') = 0·0475

P(OK, D) = 0·00465

P(OK, D') = 0·92535

b) Find the probability that a randomly selected casting

(i) was defective if we know that the sand was wet. [2]

$P(D|W) = \frac{0.005}{0.005 + 0.015} = 0.25$

(ii) was defective and the sand was wet. [2]

$P(D \cap W) = 0.005$

(iii) was defective [2]

$0.005 + 0.0025 + 0.00465$

$= 0.01215$

c) Find the probability that a randomly selected casting, which was found to be defective, had sand which was too wet.

[2]

$P(W \mid D)$

$= \dfrac{0.005}{0.01215}$

$= 0.4115$ (4 dp).

Question 11 **(3 marks)**

Consider the equation $y = 8x^3$. When x increases from 25 to 25.1, show that the approximate increase in y obtained using $\delta y = \frac{dy}{dx} \bullet \delta x$ is 1500.

$y = 8x^3$

$\frac{dy}{dx} = 24x^2$

$\delta x = 0.1$

$\Rightarrow \quad \delta y = 24x^2 \times 0.1$

$= 24(25^2) \times 0.1$

$= 1500$

Question 12 **(7 marks)**

Packets of soap powder are labelled as weighing 850 grams. However, the actual weights of the packets are normally distributed with a mean of 870 grams and standard deviation of 30 grams. $X \sim N(870, 30^2)$

a) Find the probability that a randomly chosen packet weighs less than the labelled weight.

[1]

$P(X < 850) = 0.2525$ (4 dp)

b) A consumer group is lobbying to have the following legislation introduced: if any packet weighs less than the labelled weight by more than 30 grams then it is deemed unacceptable and the manufacturers should be prosecuted.

What percentage of those packets weighing less than the labelled weight would be deemed unacceptable?

[3]

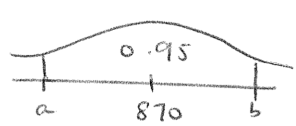
$P(X < 820) = 0.04779$ (5 dp)

$\therefore$ Percentage deemed unacceptable is:

$\frac{0.04779}{0.2525} \approx 18.9\%$ (1 dp)

c) The company claims that 95% of its packets lie within a range of weights that it would consider acceptable. Find to the nearest whole number of grams the range of weights that the company would consider acceptable.

[3]



acceptable range

$811g \leq X \leq 929g$

Question 13 **(8 marks)**

A particle moves in a straight line such that its position $s(t)$ metres from a fixed point O at time t seconds is given by the equation $s(t) = 18t - 3t^2$, where $t \geq 0$.

Find:

a) when and where its velocity is zero

[3]

$v(t) = 18 - 6t$

$18 - 6t = 0$

$t = 3\text{ s}$

$s(3) = 27\text{ m}$ $\therefore$ Velocity is zero at $t = 3\text{ s}$ when particle is 27m from O.

b) the acceleration at this time

[2]

$a(t) = -6\ \text{m/s}^2$

$a(3) = -6\ \text{m/s}^2$

c) the distance travelled in the first 4 seconds

[3]

$s(0) \to s(3)$: 27 m

$s(3) \to s(4)$: 3 m

} Distance travelled is 30m.

End of examination

Question 14

(10 marks)

The Nibbly's Nuts Company produces two different bags of fruit & nut mixes. One is sold under the company name (Nibbly's Nuts) whilst the other is sold as a 'budget' brand.

Each 1 kg bag of fruit & nut mix contains different quantities of the two ingredients, nuts and dried fruit. The table below shows these mixtures along with the cost and sell price information.

Let x represent the number of 1 kg bags of the Nibbly's Nuts mix, and
Let y represent the number of 1 kg bags of the Budget mix.

	Nuts	**Dried fruit**	**Cost per kg**	**Sell per kg**
Nibbly's Nuts (x)	600g	400g	$1.80	$2.90
Budget (y)	800g	200g	$1.50	$2.70

Each week the company has a maximum amount of 1600 kg of nuts and 920kg of dried fruit available to be processed into the 1 kg bag mixes.

The company also plans to make at least twice as many packets of the Nibbly's Nuts mix as the Budget mix.

a) Write down the three inequalities (other than $x \geq 0$ and $y \geq 0$) that represent the constraints in the above situation.

[2]

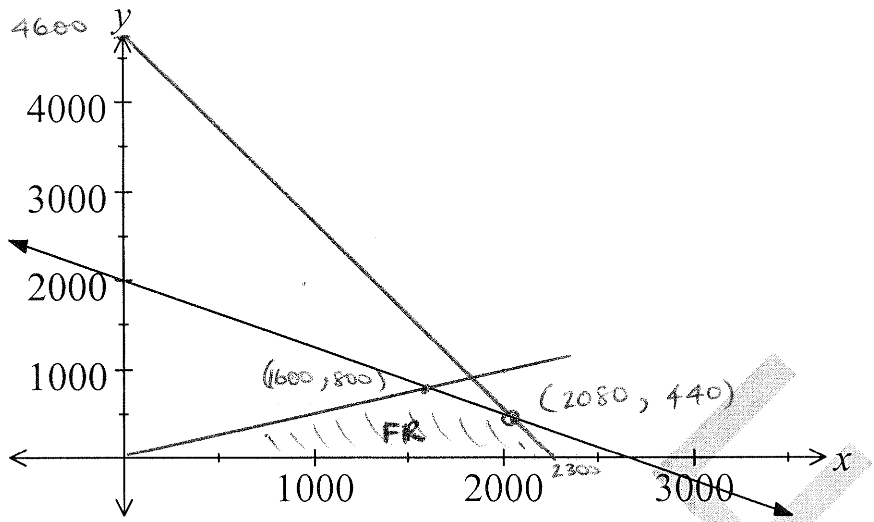
$0.6x + 0.8y \leq 1600$

$0.4x + 0.2y \leq 920$

$y \leq \frac{x}{2}$

b) One of these constraints is already on the graph below. Add the other constraints and shade in the feasible region.

[3]



c) How many 1 kg bags of each mix should the Nibbly's Nuts Company produce in order to maximise profits and what is this maximum profit?

[3]

(x, y)	$P = 1.10x + 1.20y$
1600, 800	2720
2080, 440	2816
2300, 0	2530

∴ 2080 bags of Nibbly's Nuts and 440 bags of Budget need to be produced for a max profit of $2816

d) By how much should the Nibbly's Nuts Company increase the sell price of the Budget mix before production numbers change from those found in (c)?

[2]

$P = 1.10x + ky$

$1.10(1600) + 800k = 1.10(2080) + 440k$

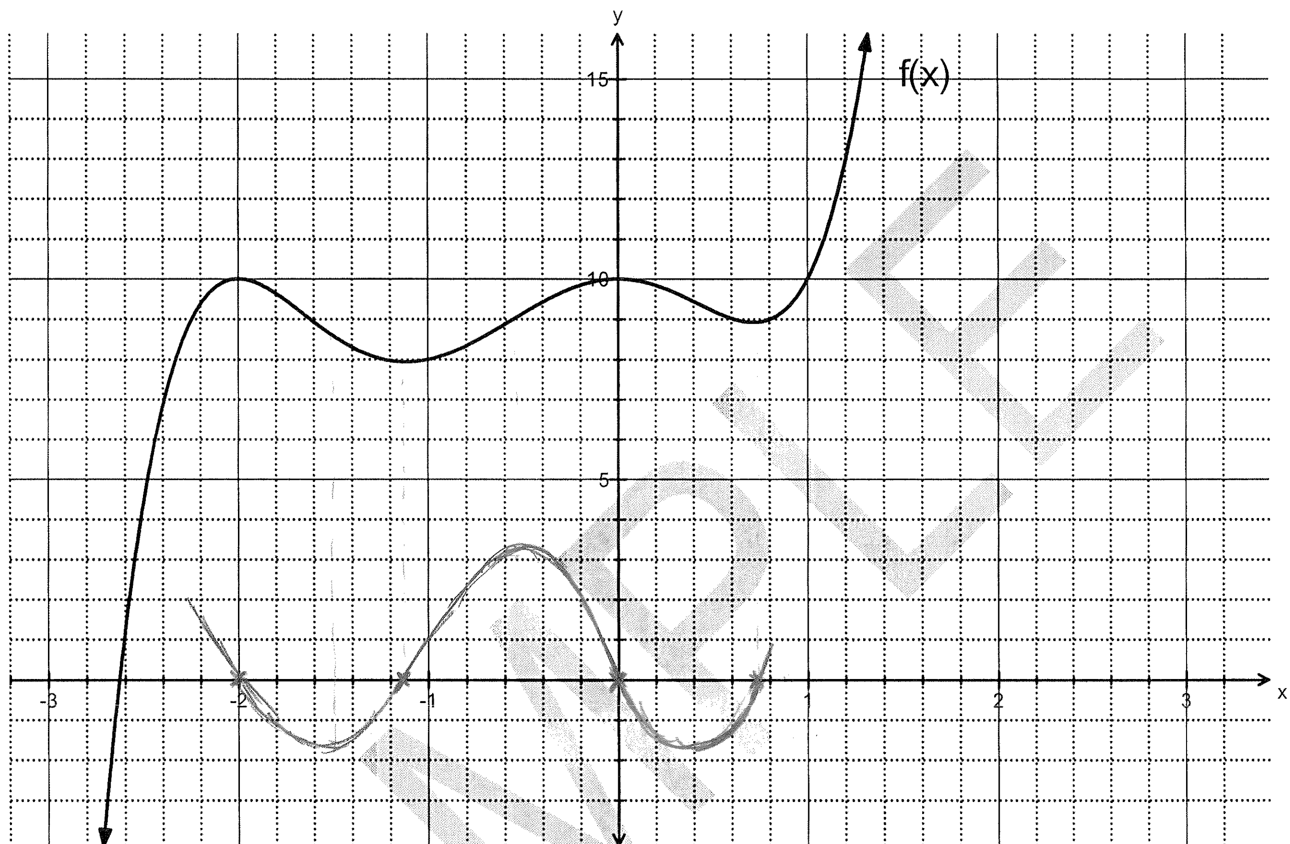
$k \approx 1.47$

A profit of > $1.47 will change solⁿ. To get a profit of > $1.47 on Budget nuts, the sale price must be $2.97 which is an increase of 27 cents.

Question 15 **(5 marks)**

a) Sketch the gradient function graph of y = f(x) on the same set of axes provided.

[3]



b) Give the exact coordinates of the points of inflection of the function $y = 2x^5 + 7x^3 - 12$

[2]

Horizontal point of Inflection

at (0, -12)

Question 16 **(5 marks)**

The car trip from Perth to Northam takes about 100 minutes. A random variable X is the number of minutes (in excess of 100) which it takes to make the trip from Perth to Northam. The probability distribution is modelled by

$$f(x) = \begin{cases} \frac{1}{20}\left(1 - \frac{x}{20}\right); & 0 \le x \le 20 \\ \frac{1}{20}\left(1 + \frac{x}{20}\right); & -20 \le x < 0 \\ 0; & otherwise \end{cases}$$

a) Show that f is a probability function

[2]

$\int_0^{20} 0.05(1 - 0.05x)\,dx = 0.5$

$\int_{-20}^{0} 0.05(1 + 0.05x)\,dx = 0.5$

Since the total area is 1, this is a probability function

b) What is the significance of the negative values of x?

[1]

Number of minutes <u>less</u> than 100 mins.

c) Determine the probability that the trip takes more than 95 minutes given that it took less than 100 minutes.

[2]

$P(x > 95 \mid x < 100) = P(x > -5 \mid x < 0)$

$= \dfrac{\int_{-5}^{0} 0.05(1 + 0.05x)\,dx}{\int_{-20}^{0} 0.05(1 + 0.05x)\,dx}$

$= 0.4375$